Topic — Homogeneous linear with constant coefficient

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(1) Homogeneous linear equation with constant coefficients :—

We have

$$(D^n + a_1 D^{n-1} + a_2 D^{n-2} + \cdots + a_n) y = f(x) \quad (1)$$

We shall now deal briefly with the corresponding equation in two variables

$$(D^n + a_1 D^{n-1} D' + a_2 D^{n-2} D'^2 + \cdots + a_n D'^n) z = f(x, y) \quad (2)$$

i.e. $\phi(D, D') = f(x, y)$ where

$$D = \frac{\partial}{\partial x} \quad \& \quad D' = \frac{\partial}{\partial y}$$

and the coefficients $a_1, a_2, \ldots, a_n$ are merely constants.

Written in full, the equation (2) takes the form

$$\frac{\partial^n z}{\partial x^n} + a_1 \frac{\partial^n z}{\partial x^{n-1} \partial y} + a_2 \frac{\partial^n z}{\partial x^{n-2} \partial y^2} + \cdots + a_n \frac{\partial^n z}{\partial y^n} = f(x, y) \quad -(3)$$

The above equation is called homogeneous since all the derivatives are of the same order and it is called linear since the derivatives appear only in first degree.

On, the other hand, when all the derivative appearing in the partial differential equation are not of the same order, then it is called a non homogeneous linear equation with constant coefficient.

The complete solution of (2) or (3) will consist of two parts.

(a) the complete function (C.F)

and (b) the particular Integral

the complementary function is the solution of $\phi(D, D')z = 0$ (R.H.S = 0)

The complementary function is the complete solution of the equation $\phi(D, D')z = 0$ which must contain n arbitrary constants. The particular